27

CURRENT AND RESISTANCE

Conceptual Questions

27.1. You can use a compass placed beside the wire and observe the compass needle change direction when the battery is disconnected and then reconnected. So something must be happening in the wire. You can feel the heat emanating from the light bulb. You can use charged glass and plastic rods to find that charge does not accumulate at the light bulb. Energy is transferred somehow from the battery to the bulb. Some kind of charge carrier must flow through the wire.

27.2. **(a)** No, either flow could explain the observations we can make. A flow of positive charges in one direction through the circuit would "look the same" as a flow of negative charges in the opposite direction in terms of the observations we can make.
(b) Yes. Connect a conducting wire between the plates of a charged parallel plate capacitor. The capacitor will discharge as charges move through the wire from one plate to the other. The experiment provides evidence that the charge is carried as discrete particles. We can recharge the capacitor plates using glass and plastic rods and rubbing charges onto or off of the plates in discrete amounts.

27.3. The density of electrons is enormous so the mean time between collisions is very, very small. A huge number of collisions can carry energy very, very quickly, seemingly instantaneously.

27.4. No. The net electric field inside the wire is zero.

27.5. Current is the rate at which charge moves through a wire. Current density, J, is the current per square meter of cross section: $J = \dfrac{I}{A}$.

27.6. $I_a = I_d > I_b = I_c$ Conservation of current insures that the current into the top junction point, I_a, must equal the current out of that point $I_b + I_c$. Similarly at the bottom junction point $I_b + I_c = I_d$. Since the wires are identical, $I_b = I_c$.

27.7. $a > b = c$. Bulbs b and c are in series, thus the potential difference across each of them and current flowing through them is smaller than for bulb a.

27.8. $a = b = c$. All three bulbs are connected directly to the positive and negative terminals of the batteries so have the same potential difference across them, thus the same current and brightness.

27.9. **(a)** $I_1 = I_2$. Due to conservation of current, the current everywhere in the wire is the same. The number of charges passing per unit time must be the same in wires 1 and 2.
(b) By definition, $J_1 = \dfrac{I}{A_1}$ and $J_2 = \dfrac{I}{A_2}$. Since $A_1 < A_2, J_1 > J_2$.

(c) Since $\sigma_1 = \sigma_2 = \sigma,\ E_1 = \dfrac{J_1}{\sigma}$ and $E_2 = \dfrac{J_2}{\sigma}$. So $E_1 > E_2$.

(d) We have $J = nev_d$. Since $J_1 > J_2,\ (v_d)_1 > (v_d)_2$.

27.10. **(a)** Since $J = \dfrac{I}{A}$, when *I* is doubled, *J* is doubled.

(b) The conduction electron density is a property of the material, so it is unchanged.

(c) The mean time between collisions τ is a property of the material and is unchanged for a constant temperature.

(d) Since $J = nev_d$ and *J* is doubled but *n* and *e* are constants, v_d is also doubled.

27.11. $R_d > R_a = R_e > R_c > R_b$. Calculate $R = \dfrac{\rho L}{A}$:

$$R_a = \frac{\rho L}{\pi r^2} \qquad R_c = \frac{\rho(2L)}{(\pi)(2r)^2} = \frac{1}{2}R_a$$

$$R_b = \frac{\rho L}{(\pi)(2r)^2} = \frac{1}{4}R_a \qquad R_d = \frac{\rho(2L)}{\pi r^2} = 2R_a$$

$$R_e = \frac{\rho(4L)}{(\pi)(2r)^2} = R_a$$

27.12. **(a)** True. The chemical reactions in the electrolytes separate the positive and negative charges. This creates a potential difference. The charges flowing in the circuit have energy due to this potential difference.

(b) False. It is true that a battery is a source of potential difference. But the potential difference is always the same ONLY for an ideal battery. In a real battery there are energy losses in the battery itself so the terminal voltage is not always the same.

(c) False. The current leaving the battery depends upon the resistance in the circuit.

Exercises and Problems

Exercises

Section 27.1 The Electron Current

27.1. **Solve:** The wire's cross-sectional area is $A = \pi r^2 = \pi(1.0\times10^{-3}\ \text{m})^2 = 3.1415\times10^{-6}\ \text{m}^2$, and the electron current through this wire is $i = \dfrac{N_e}{\Delta t} = 2.0\times10^{19}\ \text{s}^{-1}$. Using Table 27.1 for the electron density of iron and Equation 27.3, the drift velocity is

$$v_\text{d} = \frac{i}{nA} = \frac{2.0\times10^{19}\ \text{s}^{-1}}{(8.5\times10^{28}\ \text{m}^{-3})(3.1415\times10^{-6}\ \text{m}^2)} = 7.5\times10^{-5}\ \text{m/s} = 75\ \mu\text{m/s}$$

27.2. **Solve:** Using Equation 27.3 and Table 27.1, the electron current is

$$i = nAv_\text{d} = (5.9\times10^{17}\ \text{m}^{-3})\pi(0.5\times10^{-3}\ \text{m})^2(5.0\times10^{-5}\ \text{m/s}) = 2.3\times10^{18}\ \text{s}^{-1}$$

The time for 1 mole of electrons to pass through a cross section of the wire is

$$t = \frac{N_A\times 1\ \text{mole}}{i} = \frac{6.02\times10^{23}}{2.3\times10^{18}\ \text{s}^{-1}} = 2.62\times10^5\ \text{s} \approx 3.0\ \text{d}$$

Assess: The drift speed is small, and Avogadro's number is large. A time of the order of 3 days is reasonable.

27.3. Solve: Equation 27.2 is $N_e = nAv_d\Delta t$. Using Table 27.1 for the electron density, we get

$$A = \frac{\pi D^2}{4} = \frac{N_e}{nv_\text{d}\Delta t}$$

$$\Rightarrow D = \sqrt{\frac{4N_\text{e}}{\pi n v_\text{d}\Delta t}} = \sqrt{\frac{4(1.0\times10^{16})}{\pi(5.8\times10^{28}\ \text{m}^{-3})(8.0\times10^{-4}\ \text{m/s})(320\times10^{-6}\ \text{s})}} = 9.3\times10^{-4}\ \text{m} = 0.93\ \text{mm}$$

27.4. Solve: The number of electrons crossing a cross-sectional area of a wire is the electron current *i*.

$$i = nAv_d = (6.0\times10^{28}\ \text{m}^{-3})\pi\left(\frac{1.6\times10^{-3}\ \text{m}}{2}\right)^2(2.0\times10^{-4}\ \text{m/s}) = 2.4\times10^{19}\ \text{s}^{-1}$$

The electron density *n* for aluminum is taken from Table 27.1. The number of electrons passing through the cross section in one day is

$$N_\text{e} = i\Delta t = (2.4\times10^{19}\ \text{s}^{-1})(365\ \text{days})(24\ \text{hr/day})(3600\ \text{s/hr}) = 7.6\times10^{26}\ \text{electrons}$$

Assess: The large electron density compensates for the small drift velocity to deliver a huge number of electrons in current.

Section 27.2 Creating a Current

27.5. Model: Use the conduction model to relate the drift speed to the electric field strength.
Solve: From Equation 27.7, the electric field is

$$E = \frac{mv_\text{d}}{e\tau} = \frac{(9.11\times10^{-31}\ \text{kg})(2.0\times10^{-4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(5.0\times10^{-14}\ \text{s})} = 0.023\ \text{V/m}$$

27.6. Solve: (a) Each gold atom has one conduction electron. Using Avogadro's number and *n* as the number of moles, the number of atoms is

$$N = nN_\text{A} = \frac{m}{M_\text{A}}N_\text{A} = \frac{\rho V}{M_\text{A}}N_\text{A} = \frac{\rho(\pi r^2 L)}{M_\text{A}}N_A$$

The density of gold is $\rho = 19{,}300\ \text{kg/m}^3$, the atomic mass is $M_\text{A} = 197\ \text{g mol}^{-1}$, $r = 0.50\times10^{-3}\ \text{m}$, $L = 0.10\ \text{m}$, and $N_\text{A} = 6.02\times10^{23}\ \text{mol}^{-1}$. Substituting these values, we get $N = 4.6\times10^{21}$ electrons.

(b) If all the electrons are transferred a charge of $(4.6\times10^{21})(-1.60\times10^{-19}\ \text{C}) = -740\ \text{C}$ will be delivered. To deliver a charge of -32 nC, however, the electrons within a length *l* have to be delivered. Thus,

$$l = \frac{-32\times10^{-9}\ \text{C}}{-740\ \text{C}}(10\ \text{cm}) = 4.3\times10^{-10}\ \text{cm} = 4.3\times10^{-12}\ \text{m}$$

27.7. Solve: (a) The electron current is

$$i = nAv_d \Rightarrow v_d = \frac{i}{nA} = \frac{(3.5\times10^{17}\ \text{s}^{-1})}{(6.0\times10^{28}\ \text{m}^{-3})\pi(5.0\times10^{-4}\ \text{m})^2}$$
$$= 7.43\times10^{-6}\ \text{m/s}$$

The electron drift speed is 7.4×10^{-6} m/s. The electron density for aluminum is taken from Table 27.1.
(b) The electron drift velocity is related to the electric field in the wire by

$$v_d = \frac{e\tau}{m}E \Rightarrow \tau = \frac{mv_d}{eE} = \frac{(9.11\times10^{-31}\ \text{kg})(7.43\times10^{-6}\ \text{m/s})}{(1.6\times10^{-19}\ \text{C})(2.0\times10^{-3}\ \text{V/m})} = 2.1\times10^{-14}\ \text{s}$$

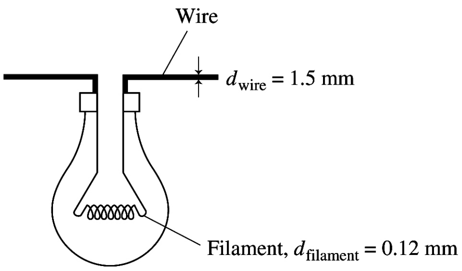
Assess: This is about the right order of magnitude based on the examples in the text.

27.8. Model: We will use the model of conduction to relate the electric field strength to the mean free time between collisions.
Solve: From Equation 27.8, the electric field is

$$i=\frac{ne\tau A}{m}E=\frac{(8.5\times10^{28}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})(4.2\times10^{-15}\ \text{s})\pi(0.9\times10^{-3}\ \text{m})^2}{(9.11\times10^{-31}\ \text{kg})}0.065\ \text{V/m}=1.6\times10^{20}\ \text{s}^{-1}$$

Section 27.3 Current and Current Density

27.9. Visualize:


Wire
d_{wire} = 1.5 mm
Filament, d_{filament} = 0.12 mm

The current density *J* in a wire, as given by Equation 27.13, does not depend on the thickness of the wire.
Solve: (a) The current in the wire is

$$I_{\text{wire}}=J_{\text{wire}}A_{\text{wire}}=(4.5\times10^5\ \text{A/m}^2)\pi\left[\frac{1}{2}(1.5\times10^{-3}\ \text{m})\right]^2=0.795\ \text{A}$$

Because current is continuous, $I_{\text{wire}}=I_{\text{filament}}$. Thus, $I_{\text{filament}}=0.975\ \text{A}\approx0.80\ \text{A}$.
(b) The current density in the filament is

$$J_{\text{filament}}=\frac{I_{\text{filament}}}{A_{\text{filament}}}=\frac{0.795\ \text{A}}{\pi\left[\frac{1}{2}(0.12\times10^{-3}\ \text{m})\right]^2}=7.0\times10^7\ \text{A/m}^2$$

27.10. Solve: (a) The current density is

$$J=\frac{I}{A}=\frac{I}{\pi R^2}=\frac{0.85\ \text{A}}{\pi\left[\frac{1}{2}(0.00025\ \text{m})\right]^2}=1.7\times10^7\ \text{A/m}^2$$

(b) The electron current, or number of electrons per second, is

$$\frac{N_{\text{e}}}{\Delta t}=\frac{I}{e}=\frac{0.85\ \text{A}}{1.60\times10^{-19}\ \text{C}}=\frac{0.85\ \text{C/s}}{1.60\times10^{-19}\ \text{C}}=5.3\times10^{18}\ \text{s}^{-1}$$

27.11. Solve: From Equation 27.13, the current in the wire is

$$I=JA=(7.50\times10^5\ \text{A/m}^2)(2.5\times10^{-6}\ \text{m}\times75\times10^{-6}\ \text{m})=0.141\ \text{mA}$$

The charge that flows in 15 min is the current times the time.

$$Q=I\Delta t=(0.141\ \text{A})(900\ \text{s})=127\ \text{C}\approx130\ \text{C}$$

27.12. Solve: Equation 27.10 is $Q=I\Delta t$. The amount of charge delivered is

$$Q=(10.0\ \text{A})\left(5.0\ \text{min}\times\frac{60\ \text{s}}{1\ \text{min}}\right)=3.0\times10^3\ \text{C}$$

The number of electrons that flow through the hair dryer is

$$N = \frac{Q}{e} = \frac{3.0\times10^{3}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 1.88\times10^{22}$$

27.13. Visualize: The relationship between current, charge, and time is $I = \Delta Q/\Delta t$.

Solve: The current is $I = \Delta Q/\Delta t = 9.0\times10^{-12}\ \text{C}/(5.0\times10^{-4}\ \text{s}) = 1.8\ \mu\text{A}$

Assess: We expect a small current due to very small flow of charge.

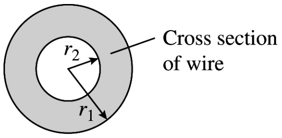
27.14. Solve: (a) The current density is

$$J = \frac{I}{A} = \frac{2.5\ \text{A}}{4.0\times10^{-6}\ \text{m}^2} = 6.2\times10^{5}\ \text{A/m}^2 = 6.3\times10^{5}\ \text{A/m}^2$$

(b) Using Equation 27.13 and Table 27.1, the drift speed is

$$v_\text{d} = \frac{J}{ne} = \frac{6.25\times10^{5}\ \text{A/m}^2}{(6.0\times10^{28}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})} = 6.5\times10^{-5}\ \text{m/s}$$

27.15. Visualize:



Solve: The current-carrying cross section of the wire is

$$A = \pi r_1^2 - \pi r_2^2 = \pi[(0.0010\ \text{m})^2 - (0.0050\ \text{m})^2] = 2.356\times10^{-6}\ \text{m}^2$$

The current density is

$$J = \frac{10\ \text{A}}{2.356\times10^{-6}\ \text{m}^2} = 4.2\times10^{6}\ \text{A/m}^2$$

27.16. Solve: The total charge in the battery is

$$Q = I\Delta t = (90\ \text{A})(3600\ \text{s}) = 3.2\times10^{5}\ \text{C}$$

Section 27.4 Conductivity and Resistivity

27.17. Model: Use the model of conduction to relate the mean time between collisions to conductivity.
Solve: From Equation 27.16, Table 27.1, and Table 27.2, the mean time between collisions for aluminum is

$$\tau_\text{Al} = \frac{m\sigma_\text{Al}}{n_\text{Al}e^2} = \frac{(9.11\times10^{-31}\ \text{kg})(3.5\times10^{7}\ \Omega^{-1}\text{m}^{-1})}{(6.0\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})^2} = 2.1\times10^{-14}\ \text{s}$$

Similarly, the mean time between collisions for iron is $\tau_\text{iron} = 4.2\times10^{-15}$ s.

Assess: The mean time between collisions in metals are of the order of 10^{-14} s.

27.18. Solve: The current density is $J = \sigma E$. Using Equation 27.17 and Table 27.2, the current in the wire is

$$I = \sigma EA = (3.5\times10^{7}\ \Omega^{-1}\text{m}^{-1})(0.012\ \text{V/m})(4\times10^{-6}\ \text{m}^2) = 1.68\ \text{A}$$

27.19. Model: Assume the battery is ideal.

Solve: (a) The electric field inside the wire is $E = \Delta V_{\text{wire}}/L$. Attaching the wire to the battery makes $\Delta V_{\text{wire}} = \Delta V_{\text{bat}} = 1.5\text{ V}$. Thus,

$$E = \frac{1.5\text{ V}}{0.15\text{ m}} = 10\text{ V/m}$$

(b) Using Table 27.2, the current density is

$$J = \sigma E = \frac{E}{\rho} = \frac{10\text{ V/m}}{1.5\times10^{-6}\ \Omega\text{ m}} = 6.7\times10^{6}\text{ A/m}^2$$

(c) The current in a wire is related to the potential difference by $I = \Delta V_{\text{wire}}/R$. Thus,

$$R = \frac{\Delta V_{\text{wire}}}{I} = \frac{1.5\text{ V}}{2\text{ A}} = 0.75\ \Omega$$

The resistance is related to the wire's geometry by

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \Rightarrow r = \sqrt{\frac{\rho L}{\pi R}} = \sqrt{\frac{(1.5\times10^{-6}\ \Omega\text{ m})(0.15\text{ m})}{\pi(0.75\ \Omega)}} = 3.1\times10^{-4}\text{ m} = 0.31\text{ mm}$$

Thus, the wire's diameter is $d = 2r = 0.62$ mm.

27.20. Solve: From Equation 27.17 and Table 27.2, the electric field is

$$E = \frac{J}{\sigma} = \frac{I}{\sigma A} = \frac{5.0\text{ A}}{(1.0\times10^{7}\ \Omega^{-1}\text{m}^{-1})\pi(1.0\times10^{-3}\text{ m})^2} = 0.16\text{ V/m}$$

27.21. Solve: From Equations 27.17 and 27.18, the resistivity is

$$\rho = \frac{E}{J} = \frac{E}{I/A} = \frac{EA}{I} = \frac{E\pi r^2}{I} = \frac{(0.0075\text{ V/m})\pi(0.50\times10^{-3}\text{ m})^2}{3.9\times10^{-3}\text{ A}} = 1.51\times10^{-6}\ \Omega\text{ m}$$

From Table 27.2, we see that the wire is made of nichrome.

27.22. Solve: From Equations 27.17 and 27.18, the resistivity is

$$\rho = \frac{E}{J} = \frac{E}{I/A} = \frac{EA}{I} = \frac{E\pi r^2}{I} = \frac{(0.085\text{ V/m})\pi(1.5\times10^{-3}\text{ m})^2}{12\text{ A}} = 5.0\times10^{-8}\ \Omega\text{ m}$$

27.23. Solve: (a) Since $J = \sigma E$ and $J = I/A$, the electric field is

$$E = \frac{I}{\sigma A} = \frac{I}{\pi r^2 \sigma} = \frac{0.020\text{ A}}{\pi(0.25\times10^{-3}\text{ m})^2(6.2\times10^{7}\ \Omega^{-1}\text{m}^{-1})} = 1.64\times10^{-3}\text{ V/m}$$

(b) Since the current density is related to v_d by $J = I/A = nev_d$, the drift speed is

$$v_{\text{d}} = \frac{I}{\pi r^2 ne} = \frac{0.020\text{ A}}{\pi(0.25\times10^{-3}\text{ m})^2(5.8\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})} = 1.10\times10^{-5}\text{ m/s}$$

Assess: The values of n and σ for silver have been taken from Table 27.1 and Table 27.2. The drift velocity is typical of metals.

27.24. Model: Because current is conserved, the currents in the two segments of the wire are the same.

Solve: The currents in the two segments of the wire are related by $I_1 = I_2 = I$. But $I = AJ$, so we have $A_1J_1 = A_2J_2$. The wire's diameter is constant, so $J_1 = J_2$ and $\sigma_1 E_1 = \sigma_2 E_2$. The ratio of the electric fields is

$$\frac{E_2}{E_1} = \frac{\sigma_1}{\sigma_2} = \frac{1}{\sigma_2/\sigma_1} = \frac{1}{2}$$

Section 27.5 Resistance and Ohm's Law

27.25. Model: Assume the battery is ideal.

Solve: (a) Because the battery provides a current of 0.50 A and the current is defined as the amount of charge that passes through a cross section per second, the charge lifted by the escalator is 0.50 C/s.

(b) The work done by the escalator in lifting charge Q is

$$W = U = Q\Delta V = (1.0\text{ C})(1.5\text{ V}) = 1.5\text{ J}$$

(c) The power output of the charge escalator is the work done by the escalator per unit time:

$$P = \frac{W}{\Delta t} = \frac{\Delta U}{\Delta t} = \frac{Q\Delta V}{\Delta t} = I\Delta V = (0.50\text{ A})(1.5\text{ V}) = 0.75\text{ W}$$

27.26. Solve: (a) Resistivity depends only on the type of material, not the geometry of the wire. Wires 1 and 2 are made of the same material, so $\rho_2 = \rho_1$ and thus $\rho_2/\rho_1 = 1.00$.

(b) The resistance of a wire of length L and radius r is given by Equation 27.22:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

Because the two wires have the same resistivity,

$$\frac{R_2}{R_1} = \frac{\rho L_2/\pi r_2^2}{\rho L_1/\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 \frac{L_2}{L_1} = \left(\frac{1}{2}\right)^2 \frac{2}{1} = \frac{1}{2} = 0.50$$

27.27. Solve: (a) Using Table 27.2 and Equation 27.22, the resistance is

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{(2.4\times10^{-8}\ \Omega\text{m})(2.0\text{ m})}{\pi(1.0\times10^{-4}\text{ m})^2} = 1.5\ \Omega$$

(b) The resistance is

$$R = \frac{\rho L}{A} = \frac{\rho L}{d^2} = \frac{(3.5\times10^{-5}\ \Omega\text{m})(0.10\text{ m})}{(0.0010\text{ m})^2} = 3.5\ \Omega$$

27.28. Model: The material is ohmic. Assume the wire has a circular cross section.

Visualize: Ohm's law implies $R = \Delta V/I$.

Solve:

$$R = \rho\frac{L}{A} \Rightarrow \rho = R\frac{A}{L} = \left(\frac{\Delta V}{I}\right)\frac{A}{L} = \left(\frac{1.5\text{ V}}{8.0\text{ A}}\right)\frac{\pi(0.33/2\text{ mm})^2}{1.0\text{ m}} = 1.6\times10^{-8}\ \Omega\cdot\text{m}$$

This value is the resistivity of silver according to the table in the textbook, so the material is probably silver.

Assess: Silver is an excellent conductor, but its cost relegates it to specialty applications in electronics.

27.29. Model: Assume the battery is an ideal battery.

Solve: Connecting the wire to the battery leads to an electric field inside the wire that is given by Equation 27.19:

$$E_{\text{wire}} = \frac{\Delta V_{\text{wire}}}{L} \Rightarrow \Delta V_{\text{wire}} = LE_{\text{wire}} = (0.30\text{ m})(5.0\times10^{-13}\text{ V/m}) = 1.5\times10^{-13}\text{ V} = 1.5\text{ mV}$$

27.30. Model: Assume the battery is an ideal battery.

Solve: (a) The current in a wire is related to its resistance R and to the potential difference ΔV_{wire} between the ends of the wire as

$$I = \frac{\Delta V_{\text{wire}}}{R} \Rightarrow R = \frac{\Delta V_{\text{wire}}}{I} = \frac{1.5\text{ V}}{0.50\text{ A}} = 3.0\ \Omega$$

From Equation 27.22,

$$R=\frac{\rho L}{A}\Rightarrow L=\frac{RA}{\rho}=\frac{(3.0\,\Omega)\pi(0.30\times10^{-3}\text{ m})^2}{2.8\times10^{-8}\,\Omega\text{m}}=30\text{ m}$$

(b) For a uniform wire of the same material, $R\propto L$. If the wire in this problem is cut in half, its resistance will decrease to $\frac{1}{2}(3\,\Omega)=1.5\,\Omega$. Thus, using $I=\Delta V_{\text{wire}}/R$, we have $I=1.0$ A.

27.31. Model: Assume the battery is an ideal battery.
Solve: We can find the current I from Equation 27.23 provided we know the resistance R of the gold wire. From Equation 27.22 and Table 27.2, the resistance of the wire is

$$R=\frac{\rho L}{A}=\frac{\rho L}{\pi r^2}=\frac{(2.4\times10^{-8}\,\Omega\text{m})(100\text{ m})}{\pi(0.050\times10^{-3}\text{ m})^2}=305.6\ \Omega\Rightarrow I=\frac{\Delta V_{\text{wire}}}{R}=\frac{0.70\text{ V}}{305.6\ \Omega}=2.3\text{ mA}$$

27.32. Model: The resistance of a “resistor” (the blood-filled artery in this case) is given by . $R=L/\sigma A$.
We are asked for the resistances of the blood itself, so we ignore the artery walls.
Visualize: We are given $L=0.20$ m and $A=\pi r^2=\pi(d/2)^2=\pi(0.0050\text{ m})^2=7.85\times10^{-5}\text{ m}^2$.
Solve:

$$R=\frac{L}{\sigma A}=\frac{(0.20\text{ m})}{(0.63\,\Omega^{-1}\cdot\text{m}^{-1})(7.85\times10^{-5}\text{ m}^2)}=4100\,\Omega$$

Assess: $4100\,\Omega$ is a reasonable value of resistance, neither extremely large nor extremely small. The value of the conductivity of blood is also neither extremely large nor extremely small, so we are satisfied with our answer.

27.33. Solve: From Table 27.2, the resistivity of carbon is $\rho=3.5\times10^{-5}\ \Omega$ m. From Equation 27.22, the resistance of lead from a mechanical pencil is

$$R=\frac{\rho L}{A}=\frac{\rho L}{\pi r^2}=\frac{(3.5\times10^{-5}\ \Omega\text{ m})(0.060\text{ m})}{\pi(0.35\times10^{-3}\text{ m})^2}=5.5\ \Omega$$

Now apply Ohm’s law:

$$I=\frac{\Delta V}{R}=\frac{9.0\text{ V}}{5.5\ \Omega}=1.6\text{ A}$$

Assess: 1.6 A is a pretty good current. The lead might warm up.

27.34. Solve: From Table 27.2, the resistivity of aluminum is $\rho=2.8\times10^{-8}\ \Omega$ m. From Equation 27.22, the length L of a wire with a cross-sectional area A and having a resistance R is

$$L=\frac{AR}{\rho}=\frac{(10\times10^{-6}\text{ m})^2(1000\ \Omega)}{2.8\times10^{-8}\ \Omega\text{ m}}=3.57\text{ m}$$

The number of turns is the length of the wire divided by the circumference of one turn. Thus,

$$\frac{3.57\text{ m}}{2\pi(1.5\times10^{-3}\text{ m})}=380$$

27.35. Solve: The slope of the I versus ΔV graph, according to Ohm’s law, is the inverse of the resistance R. From the graph in Figure EX27.35, the slope of the material is

$$\frac{\Delta V}{I}=\frac{100\text{ V}}{2\text{ A}}=\ 50\ \Omega$$

27.36. Solve: We need an aluminum wire whose resistance and length are the same as that of a 0.50-mm-diameter copper wire. That is,

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}} L}{A_{\text{Cu}}} = R_{\text{Al}} = \frac{\rho_{\text{Al}} L}{A_{\text{Al}}} \Rightarrow A_{\text{Al}} = \left(\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}\right) A_{\text{Cu}} \Rightarrow \pi r_{\text{Al}}^2 = \left(\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}\right) \pi r_{\text{Cu}}^2$$

$$\Rightarrow r_{\text{Al}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} r_{\text{Cu}} = \sqrt{\frac{2.8\times 10^{-8}\ \Omega\ \text{m}}{1.7\times 10^{-8}\ \Omega\ \text{m}}}(0.25\ \text{mm}) = 0.32\ \text{mm}$$

We need a 0.64-mm-diameter aluminum wire.

27.37. Model: The copper wire is uniform.
Solve: Equation 27.21 relates the current in a wire to the potential difference across it:

$$I = \frac{A}{\rho L}\Delta V \Rightarrow \Delta V = \frac{\rho L I}{A} = \frac{(1.7\times 10^{-8}\ \Omega\text{m})(20\ \text{m})(8.0\ \text{A})}{\pi(1.0\times 10^{-3}\ \text{m})^2} = 0.87\ \text{V}$$

The resistivity ρ of copper is taken from Table 27.2.

Assess: While voltage drops in household wires are small compared to the applied voltage, voltage drops in transmission wires between homes and power plants could be quite large. The power company transports energy in a way that minimizes the voltage drop, as we will learn in a later chapter.

27.38. Model: The material in the battery is ohmic. Assume the wire has no resistance.
Visualize: Ohm's law implies $R = \Delta V/I$.
Solve:

$$R = \frac{\Delta V}{I} = \frac{9.0\ \text{V}}{21\ \text{A}} = 0.43\ \Omega$$

Assess: Were it not for this internal resistance shorted batteries could melt wires.

Problems

27.39. Solve: Equation 27.17 will be used to relate electric field strength with the diameter. We have

$$J = \frac{I}{A} = \frac{I}{\frac{1}{4}\pi D^2} = \frac{4I}{\pi D^2} = \sigma E \Rightarrow I = \frac{\sigma \pi E D^2}{4}$$

Because the current is the same in the two wires,

$$E_{\text{nichrome}} = \left(\frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}}\right)\left(\frac{D_{\text{aluminum}}}{D_{\text{nichrome}}}\right)^2 E_{\text{aluminum}}$$

Using the values of σ from Table 27.2,

$$E_{\text{nichrome}} = \left(\frac{3.5\times 10^7\ \Omega^{-1}\ \text{m}^{-1}}{6.7\times 10^5\ \Omega^{-1}\ \text{m}^{-1}}\right)\left(\frac{1.0\ \text{mm}}{2.0\ \text{mm}}\right)^2 (0.0080\ \text{V/m}) = 0.104\ \text{V/m} \approx 0.10\ \text{V/m}$$

27.40. Solve: (a) The moving electrons are a current, even though they're not confined inside a wire. The electron current is

$$\frac{N_{\text{e}}}{\Delta t} = \frac{I}{e} = \frac{50\times 10^{-6}\ \text{A}}{1.60\times 10^{-19}\ \text{C}} = 3.1\times 10^{14}\ \text{s}^{-1}$$

This means during the time interval $\Delta t = 1$ s, 3.1×10^{14} electrons strike the screen.

(b) The current density is

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{50\times10^{-6}\ \text{A}}{\pi(0.00020\ \text{m})^2} = 4.0\times10^2\ \text{A/m}^2$$

(c) The acceleration can be found from kinematics:

$$v_1^2 = (4.0\times10^7\ \text{m/s})^2 = v_0^2 + 2a\Delta x = 2a\Delta x \Rightarrow a = \frac{(4.0\times10^7\ \text{m/s})^2}{2(5.0\times10^{-3}\ \text{m})} = 1.60\times10^{17}\ \text{m/s}^2$$

But the acceleration is $a = F/m = eE/m.$ Consequently, the electric field must be

$$E = \frac{ma}{e} = \frac{(9.11\times10^{-31}\ \text{kg})(1.60\times10^{17}\ \text{m/s}^2)}{1.6\times10^{-19}\ \text{C}} = 9.1\times10^5\ \text{V/m}$$

(d) When they strike the screen, each electron has a kinetic energy

$$K = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(9.11\times10^{-31}\ \text{kg})(4.0\times10^7\ \text{m/s}) = 7.288\times10^{-16}\ \text{J}$$

Power is the *rate* at which the screen absorbs this energy. The power of the beam is

$$P = \frac{\Delta E}{\Delta t} = K\frac{N_\text{e}}{\Delta t} = (7.288\times10^{-16}\ \text{J})(3.1\times10^{14}\ \text{s}^{-1}) = 0.23\ \text{J/s} = 0.23\ \text{W}$$

Assess: Power delivered to the screen by the electron beam is reasonable because the screen over time becomes a little warm.

27.41. Model: Assume each proton frees 35,000 electrons.
Visualize: The current before magnification is what comes from the proton collisions, so we put in our calculations $3.5\ \mu\text{A}/100 = (3.5\ \mu\text{C}/100)/\text{s}$.
Solve:

$$\frac{\#\text{protons}}{\text{s}} = \left(\frac{1\ \text{proton}}{35{,}000\ \text{electrons}}\right)\left(\frac{1\ \text{electron}}{1\ e}\right)\left(\frac{1\ e}{1.6\times10^{-19}\ \text{C}}\right)\left(\frac{(3.5\ \mu\text{C})/100}{\text{s}}\right) = 6.2\times10^6$$

So 6.2×10^6 protons hit the detector every second.
Assess: That seems like a lot of protons, but it produces a small current that must be amplified.

27.42. Solve: (a) The current associated with the moving film is the rate at which the charge on the film moves past a certain point. The tangential speed of the film is

$$v = \omega r = (90\ \text{rpm})(4.0\ \text{cm}) = 90\frac{\text{rev}}{\text{min}}\times\frac{1\ \text{min}}{60\ \text{s}}\times\frac{2\pi\ \text{rad}}{1\ \text{rev}}\times1.0\ \text{cm} = 9.425\ \text{cm/s}$$

In 1.0 s the film moves a distance of 9.425 cm. This means the area of the film that moves to the right in 1.0 s is $(9.425\ \text{cm})(4.0\ \text{cm}) = 37.7\ \text{cm}^2$. The amount of charge that passes to the right in 1.0 s is

$$Q = (37.7\ \text{cm}^2)(-2.0\times10^{-9}\ \text{C/cm}^2) = -75.4\times10^{-9}\ \text{C}$$

Since $I = Q/\Delta t,$ we have

$$I = \frac{\left|-(75.4\times10^{-9}\ \text{C})\right|}{1\ \text{s}} = 75.4\ \text{nA}$$

The current is 75 nA.
(b) Having found the current in part (a), we can once again use $I = Q/\Delta t$ to obtain Δt:

$$\Delta t = \frac{Q}{I} = \frac{\left|-10\times10^{-6}\ \text{C}\right|}{75.4\times10^{-9}\ \text{A}} = 133\ \text{s} \approx 130\ \text{s}$$

27.43. **Model:** The current is the rate at which the charge of the ions moves through the ionic solution.
Solve: Because the atomic mass of gold is 197 g, the number of gold atoms is

$$N = \frac{M}{M_A} N_A = \left(\frac{0.50 \text{ g}}{197 \text{ g mol}^{-1}}\right) 6.02\times10^{23} \text{ mol}^{-1} = 1.53\times10^{21} \text{ atoms}$$

We need to deposit $N = 1.53\times10^{21}$ gold ions, each with a charge of -1.60×10^{-19} C, in 3.0 hours on the statue. The current is

$$I = \frac{Q}{\Delta t} = \frac{(1.53\times10^{21})(1.60\times10^{-19} \text{ C})}{3.0\times3600 \text{ s}} = 23 \text{ mA}$$

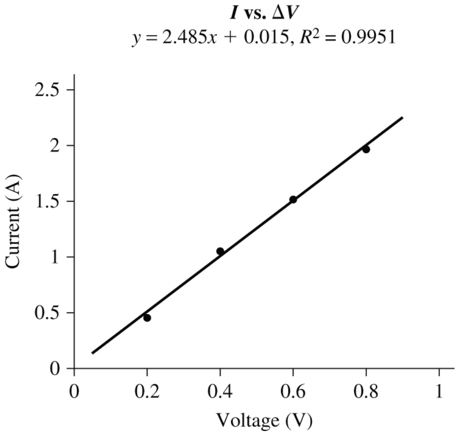
27.44. **Model:** Current is the rate at which the charge moves across a certain cross section.
Solve: **(a)** For the circular motion of the electron around the proton, Coulomb's force between the electron and the proton causes the centripetal acceleration. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{|-e|e}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 \Rightarrow \frac{1}{T} = f = \sqrt{\frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})^2}{4\pi^2(0.053\times10^{-9} \text{ m})^3(9.11\times10^{-31} \text{ kg})}} = 6.56\times10^{15} \text{ Hz}$$

The frequency is 6.6×10^{15} Hz. **(b)** Charge $Q = e$ passes any point on the orbit once every period. Thus the effective current is

$$I = \frac{Q}{\Delta t} = \frac{e}{T} = ef = (1.6\times10^{-19} \text{ C})(6.56\times10^{15} \text{ Hz}) = 1.05\times10^{-3} \text{ A}$$

27.45. **Model:** We will not initially assume the material is ohmic; this is what we are trying to find out.
Visualize: If an object (resistor) is ohmic then a graph of I vs. ΔV will be linear and the slope will be $1/R$. The cross-sectional area $A = (1.0\times10^{-3} \text{ m})\times(0.50\times10^{-3} \text{ m}) = 0.50\times10^{-6} \text{ m}^2$.
Solve:



We see from the graph that the linear fit is excellent and the slope is 2.485 A/V. The intercept is also very small, which agrees with our hypothesis that the two variables are directly proportional (and linearly related), so we proceed to compute the conductivity. From $R = \rho L/A$ and $\sigma = 1/\rho$ we get

$$\text{slope} = \frac{1}{R} = \frac{\sigma A}{L}$$

Solve for σ:

$$\sigma = \frac{(\text{slope})(L)}{A} = \frac{(2.485 \text{ A/V})(45\times10^{-3} \text{ m})}{0.50\times10^{-6} \text{ m}^2} = 2.2\times10^5 \ \Omega^{-1}\text{m}^{-1}$$

Assess: The material is ohmic, but not a particularly great conductor since the conductivity is a couple of orders less than the metal conductors in the table.

27.46. Solve: (a) A current of 1.8 pA for the potassium ions means that a charge of 1.8 pC flows through the potassium ion channel per second. The number of potassium ions that pass through the ion channel per second is

$$\frac{1.8\times10^{-12}\ \text{C/s}}{1.6\times10^{-19}\ \text{C}}=1.125\times10^{7}\ \text{s}^{-1}$$

Since the channel opens only for 1.0 ms, the total number of potassium ions that pass through the channel is $(1.125\times10^{7}\ \text{s}^{-1})(1.0\times10^{-3}\ \text{s})=1.13\times10^{4}$ atoms.

(b) The current density in the ion channel is

$$J=\frac{I}{A}=\frac{1.8\ \text{pA}}{\pi(0.30\ \text{nm}/2)^2}=\frac{1.8\times10^{-12}\ \text{A}}{\pi(0.15\times10^{-9}\ \text{m})^2}=2.5\times10^{7}\ \text{A/m}^2$$

27.47. Solve: (a) Current is defined as $I = Q/\Delta t$, so the charge delivered in time Δt is

$$Q=I\Delta t=(150\ \text{A})(0.80\ \text{s})=120\ \text{C}$$

(b) The drift speed is

$$v_\text{d}=\frac{J}{ne}=\frac{I/A}{ne}=\frac{I}{\pi r^2 ne}=\frac{150\ \text{A}}{\pi(0.0025\ \text{m})^2(8.5\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})}=5.617\times10^{-4}\ \text{m/s}$$

At this speed the electrons drift a distance

$$d=(5.617\times10^{-4}\ \text{m/s})(0.80\ \text{s})=4.49\times10^{-4}\ \text{m}=0.45\ \text{mm}$$

27.48. Model: We want the drift speed to be $v_\text{d}=60\ \text{mph}=26.8\ \text{m/s}$.

Visualize: Use $v_\text{d}=e\tau E/m$. We are given $L=1.5\ \text{m}$ and $\Delta V=9.0\ \text{V}$. The electron density of copper is $n_\text{e}=8.5\times10^{28}$.

Solve:

$$\sigma=\frac{n_\text{e}e^2\tau}{m}=n_\text{e}e\left(\frac{e\tau}{m}\right)=n_\text{e}e\frac{v_\text{d}}{E}=n_\text{e}\ e\ v_\text{d}\frac{L}{\Delta V}$$
$$=(8.5\times10^{28})e\left(\frac{1.6\times10^{-19}\ \text{C}}{e}\right)(26.8\ \text{m/s})\frac{1.5\ \text{m}}{9.0\ \text{V}}=6.1\times10^{10}\ \Omega^{-1}\text{m}^{-1}$$

This is 1000 times greater than the conductivity of copper (which is $6.0\times10^{7}\ \Omega^{-1}\text{m}^{-1}$).

Assess: We are not likely to find a material with the n_e of copper and a conductivity 1000 times that of copper.

27.49. Model: The resistance of a wire is $R=\rho L/A$.

Visualize: We are given $\rho=\rho_0[1+\alpha(T-T_0)]$. The temperature coefficient of resistivity for copper is $\alpha=3.9\times10^{-3}\ {}^\circ\text{C}^{-1}$. Because L and A don't change, $\rho/\rho_0=R/R_0$.

Solve: Solve the equation for T.

$$\rho=\rho_0[1+\alpha(T-T_0)]\Rightarrow\frac{\rho}{\rho_0}=1+\alpha(T-T_0)\Rightarrow\frac{\rho}{\rho_0}-1=\alpha(T-T_0)\Rightarrow\frac{\frac{\rho}{\rho_0}-1}{\alpha}=T-T_0\Rightarrow$$
$$T=\frac{\frac{\rho}{\rho_0}-1}{\alpha}+T_0=\frac{\frac{R}{R_0}-1}{\alpha}+T_0=\frac{\frac{0.30\ \Omega}{0.25\ \Omega}-1}{3.9\times10^{-3}\ {}^\circ\text{C}^{-1}}+20^\circ\text{C}=71^\circ\text{C}$$

Assess: We expected the temperature to be hotter since the resistance increased.

27.50. Model: We assume that Ohm's law applies to the situation.

$$I = \frac{\Delta V}{R}$$

We also use Equation 27.22, which gives R in terms of ρ, L, and A.

$$R = \frac{\rho L}{A}$$

Visualize: We are given that $\Delta V = 9.0\text{ V}$, $L = 0.050\text{ m}$, $I = 230\ \mu\text{A}$, and $A = \pi r^2 = \pi(d/2)^2 = \pi(1.5\text{ mm}/2)^2 = 1.77\times10^{-6}\text{ m}^2$.

Solve: Combine the two previous equations.

$$\rho = \frac{RA}{L} = \frac{\Delta V}{I}\frac{A}{L} = \frac{(9.0\text{ V})(1.77\times10^{-6}\text{ m}^2)}{(230\times10^{-6}\text{A})(0.050\text{ m})} = 1.4\ \Omega\cdot\text{m}$$

Assess: This resistivity is close to the value for blood (1.6 Ω · m) found in references.

27.51. Model: The resistance of a wire is related to its resistivity, length, and cross-sectional area by $R = \rho L/A$. The area of a wire is related to its diameter by $A = \pi d^2/4$. The potential drop across a resistor is related to the resistance of the resistor and the current through the resistor by $\Delta V = IR$.

Solve: (a) The resistance between the hands is

$$R = \rho L/A = \rho L/(\pi d^2/4) = 4\rho L/(\pi d^2) = 4(5.0\ \Omega\cdot\text{m})(1.6\text{ m})/(\pi(0.10\text{ m})^2) = 1.0\times10^3\ \Omega$$

(b) The potential difference across a resistor of this size when there is a current of 100 mA is

$$\Delta V = IR = (0.10\text{ A})(1.02\times10^3\ \Omega) = 100\text{ V}$$

Assess: This result tells us that a potential difference of 100 V can cause lethal currents when your skin is wet. This should give you new respect for the potential difference at all of your electrical outlets.

27.52. Model: Assume both muscle and fat are ohmic.

Visualize: The resistivity of the whole leg will be a weighted average of the resistivities of muscle and fat.

Solve:

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{\frac{\rho L}{A}} = \frac{A\Delta V}{(L)(0.82\rho_{\text{muscle}} + 0.18\rho_{\text{fat}})} = \frac{\pi(0.060\text{ m})^2(1.5\text{ V})}{(0.40\text{ m})[0.82(13\ \Omega\text{m}) + 0.18(25\ \Omega\text{m})]} = 2.8\text{ mA}$$

Assess: This is not the same as treating the system as two resistors (one made of muscle and the other of fat) in parallel; the muscle and fat are mixed together, not separate side by side.

27.53. Model: The resistance of a wire is $R = \rho L/A$.

Visualize: We are given $\rho = \rho_0[1 + \alpha(T - T_0)]$. The temperature coefficient of resistivity for copper is $\alpha = 3.9\times10^{-3}\ {}^\circ\text{C}^{-1}$. The binomial approximation is $(1+x)^n \approx 1 + nx$ for $x \ll 1$.

Solve: (a)

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{\rho\frac{L}{A}} = \frac{(\Delta V)A}{\rho_0[1+\alpha(T-T_0)]L} = \frac{(\Delta V)A[1+\alpha(T-T_0)]^{-1}}{\rho_0 L} \approx \frac{(\Delta V)A[1-\alpha(T-T_0)]}{\rho_0 L}$$

(b) Plug in the given values. For T = 20°C, $(T - T_0) = 0$.

$$I = \frac{(\Delta V)A[1-\alpha(T-T_0)]}{\rho_0 L} = \frac{(1.5\text{ V})(\pi(0.20\text{ mm})^2)[1-0]}{(1.7\times10^{-8}\ \Omega\cdot\text{m})(2.5\text{ m})} = 4.4\text{ A}$$

(c) Find the derivative of *I* with respect to *T*.

$$I=\frac{(\Delta V)A[1-\alpha(T-T_0)]}{\rho_0 L}\Rightarrow\frac{dI}{dT}=\frac{(\Delta V)A(-\alpha)}{\rho_0 L}=\frac{(1.5\text{ V})(\pi(0.20\text{ mm})^2)(-3.9\times10^{-3}\ {}^\circ\text{C}^{-1})}{(1.7\times10^{-8}\ \Omega\cdot\text{m})(2.5\text{ m})}=-0.017\text{ A/}^\circ\text{C}$$

Assess: Both results depend on $\Delta V, A, L, \rho_0,$ so changing any of those would change the answer.

27.54. Model: The conductance of a wire is $G=\sigma A/L.$

Visualize: Note that the conductance is the inverse of the resistance: $G=1/R.$

Solve: (a)

$$\Delta V=IR\Rightarrow\Delta V=I/G$$

(b) Plug in the given values.

$$G=\sigma A/L=(1.8\times10^{7}\ \Omega^{-1}\text{m}^{-1})(\pi(0.15/2\text{ mm})^2)/0.054\text{ m}=5.9\ \Omega^{-1}$$

(c) Combine parts (a) and (b).

$$\Delta V=I/G=1.5\text{ A}/5.9\ \Omega^{-1}=0.25\text{ V}$$

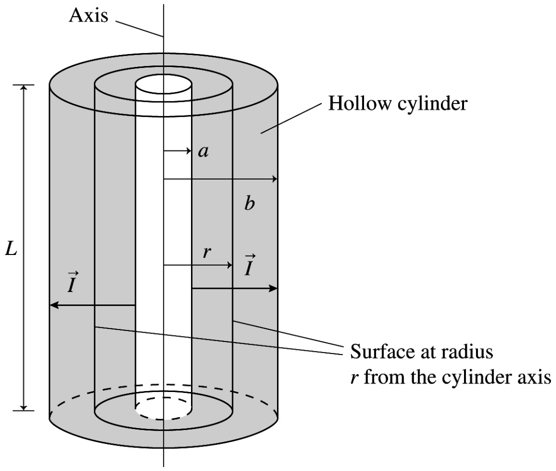
Assess: This seems like a reasonable voltage given the values of the other variables.

27.55. Solve: Equation 27.13 defines the current density as $J=I/A.$ This means

$$A=\frac{\pi D^2}{4}=\frac{I}{J}\Rightarrow D=\sqrt{\frac{4I}{\pi J}}=\sqrt{\frac{4(1.0\text{ A})}{\pi(500\text{ A/cm}^2)}}=0.050\text{ cm}=0.50\text{ mm}$$

Assess: Fuse wires are usually thin.

27.56. Visualize:



Solve: (a) Consider a cylindrical surface inside the metal at a radial distance *r* from the center. The current is flowing through the walls of this cylinder, which have surface area $A=(2\pi r)L.$ Thus

$$I=JA=\sigma E(2\pi r)L$$

Thus the electric field strength at radius *r* is

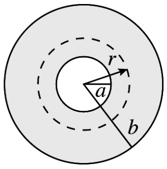
$$E=\frac{I}{2\pi\sigma Lr}$$

(b) For iron, with $\sigma = 1.0\times10^{7}\ \Omega^{-1}\ \text{m}^{-1}$,

$$E_{\text{inner}} = \left(\frac{25\ \text{A}}{2\pi(0.10\ \text{m})(1.0\times10^{7}\ \Omega^{-1}\text{m}^{-1})}\right)\left(\frac{1}{0.010\ \text{m}}\right) = 4.0\times10^{-4}\ \text{V/m}$$

$$E_{\text{outer}} = \left(\frac{25\ \text{A}}{2\pi(0.10\ \text{m})(1.0\times10^{7}\ \Omega^{-1}\text{m}^{-1})}\right)\left(\frac{1}{0.025\ \text{m}}\right) = 1.59\times10^{-4}\ \text{V/m}.$$

27.57. Visualize:



Hollow sphere

Solve: (a) Consider a spherical surface inside the hollow sphere at a radial distance r from the center. The current is flowing outward through this surface, which has surface area $A = 4\pi r^2$. Thus

$$I = JA = \sigma E(4\pi r^2)$$

Thus the electric field strength at radius r is

$$E = \frac{I}{4\pi\sigma r^2}$$

(b) For copper, with $\sigma = 1.0\times10^{7}\ \Omega^{-1}\ \text{m}^{-1}$,

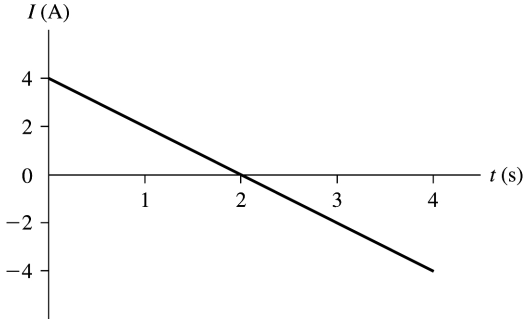
$$E_{\text{inner}} = \frac{25\ \text{A}}{4\pi(6.0\times10^{7}\ \Omega^{-1}\ \text{m}^{-1})}\frac{1}{(0.010\ \text{m})^2} = 3.3\times10^{-4}\ \text{V/m}$$

$$E_{\text{outer}} = \frac{25\ \text{A}}{4\pi(6.0\times10^{7}\ \Omega^{-1}\ \text{m}^{-1})}\frac{1}{(0.025\ \text{m})^2} = 5.3\times10^{-5}\ \text{N/C}$$

27.58. Solve: (a) Since $I = \Delta Q/\Delta t$, for infinitesimal changes

$$I = \frac{dQ}{dt} = \frac{d}{dt}(4t - t^2) = 4 - 2t$$

(b)

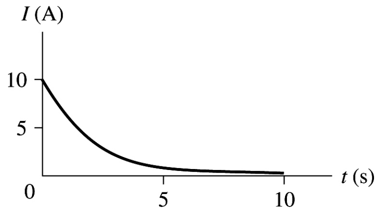


27.59. Solve: (a) Since $I = \Delta Q/\Delta t$, for infinitesimal changes

$$I = \frac{dQ}{dt} = \frac{d}{dt}(20\ \text{C})(1 - e^{-t/2.0\ \text{s}}) = (20\ \text{C})(-e^{-t/2.0\ \text{s}})(-1/2.0\ \text{s}) = (10\ \text{A})e^{-t/2.0\ \text{s}}$$

(b) The maximum value of the current occurs at $t = 0$ s and is 10 A.
(c) The values of I (A) for selected values of t are

***t* (s)**	0	1	2	4	6	8	10
***I* (A)**	10	6.07	3.68	1.35	0.50	0.18	0.07

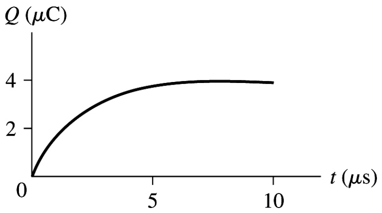


27.60. Solve: (a) Because $I = dQ/dt$,

$$Q = \int I\,dt = \int_0^t (2.0\text{ A})e^{-t/(2.0\,\mu\text{s})}dt = \left[(-4.0\text{ A }\mu\text{s})e^{-t/2.0\,\mu\text{s}}\right]_0^t = (4.0\,\mu\text{C})\left[1 - e^{-t/2.0\,\mu\text{s}}\right]$$

where we have used the condition $Q = 0$ C at $t = 0$ μs.
(b) The values of Q (μC) for selected values of t are

***t* (μs)**	0	1	2	4	6	8	10
***Q* (μC)**	0	1.57	2.52	3.46	3.80	3.92	3.98



27.61. Solve: The total charge delivered by the battery is

$$Q = \int dQ = \int_{0\text{ s}}^{\infty} I\,dt = \int_{0\text{ s}}^{\infty} (0.75\text{ A})e^{-t/(6\text{ hours})} = (0.75\text{ A})(-6\text{ hours})\left[e^{-t/(6\text{ hours})}\right]_0^{\infty}$$

$$= (0.75\text{ A})(6 \times 3600\text{ s}) = 1.62 \times 10^4\text{ C}$$

The number of electrons transported is

$$\frac{1.62 \times 10^4\text{ C}}{1.6 \times 10^{-19}\text{ C}} = 1.01 \times 10^{23}$$

27.62. Model: Because current is conserved, the current flowing in the 2.0-mm-diameter segment of the wire is the same as in the 1.0-mm-diameter segment.
Visualize: We will denote all quantities for the 1.0-mm-diameter wire with the subscript 1, and all quantities for the 2.0-mm-diameter wire with the subscript 2.
Solve: Equation 27.13 is $J = nev_\text{d}$. This means the current densities in the two segments are

$$J_1 = nev_{\text{d}1} \qquad J_2 = nev_{\text{d}2}$$

Dividing these equations, we get $v_{\text{d}2} = (J_2/J_1)v_{\text{d}1}$. Because current is conserved, $I_1 = I_2 = 2.0$ A. So,

$$\frac{J_2}{J_1} = \frac{I_2/A_2}{I_1/A_1} = \frac{A_1}{A_2} \Rightarrow v_{\text{d}2} = \frac{A_1}{A_2}v_{\text{d}1} = \left(\frac{D_1}{D_2}\right)^2 v_{\text{d}1} = \left(\frac{1.0\text{ mm}}{2.0\text{ mm}}\right)^2 (2.0 \times 10^{-4}\text{ m/s}) = 5.0 \times 10^{-5}\text{ m/s}$$

Assess: A drift velocity that is small and only $\left(\frac{1}{4}\right)$ of the drift velocity in the 1.0-mm-diameter wire is reasonable.

27.63. Model: Because current is conserved, the currents in the aluminum and the nichrome segments of the wire are the same.

Solve: From Equations 27.13 and 27.17 we have $J = I/A = \sigma E$. This means

$$E = \frac{I}{A\sigma} \Rightarrow \frac{E_{\text{nichrome}}}{E_{\text{aluminum}}} = \left(\frac{I_{\text{nichrome}}}{I_{\text{aluminum}}}\right)\left(\frac{A_{\text{aluminum}}}{A_{\text{nichrome}}}\right)\left(\frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}}\right) = \left(\frac{D^2{}_{\text{aluminum}}}{D^2{}_{\text{nichrome}}}\right)\left(\frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}}\right)$$

where we used the conservation of current and $A = \pi/4\,D^2$. For $E_{\text{nichrome}} = E_{\text{aluminum}}$, the above equation simplifies to

$$D_{\text{nichrome}} = \sqrt{\frac{\sigma_{\text{aluminum}}}{\sigma_{\text{nichrome}}}}D_{\text{aluminum}} = \sqrt{\frac{3.5\times10^7\ \Omega^{-1}\text{m}^{-1}}{6.7\times10^5\ \Omega^{-1}\text{m}^{-1}}}(1.0\ \text{mm}) = 7.2\ \text{mm}$$

27.64. Model: Electric current is conserved.

Visualize: For the top, middle, and bottom segments the subscripts “top,” “mid,” and “bot” are used.

Solve: (a) Since current is conserved, $I_{\text{top}} = I_{\text{mid}} = I_{\text{bot}} = 10\ \text{A}$.

(b) The current density is $J = I/A = I/\pi R^2$. Thus,

$$J_{\text{top}} = J_{\text{bot}} = \frac{10\ \text{A}}{\pi(0.001\ \text{m})^2} = 3.18\times10^6\ \text{A/m}^2 \qquad J_{\text{mid}} = \frac{10\ \text{A}}{\pi(0.0005\ \text{m})^2} = 1.27\times10^7\ \text{A/m}^2$$

(c) The electric field is $E = J/\sigma$. Thus,

$$E_{\text{top}} = E_{\text{bot}} = \frac{J_{\text{top}}}{\sigma} = \frac{3.18\times10^6\ \text{A/m}^2}{3.5\times10^7\ \Omega^{-1}\text{m}^{-1}} = 0.0909\ \text{V/m} \qquad E_{\text{mid}} = \frac{J_{\text{mid}}}{\sigma} = \frac{1.27\times10^7\ \text{A/m}^2}{3.5\times10^7\ \Omega^{-1}\text{m}^{-1}} = 0.364\ \text{V/m}$$

(d) The drift speed is $v_{\text{d}} = J/ne$. Thus

$$(v_{\text{d}})_{\text{top}} = (v_{\text{d}})_{\text{bot}} = \frac{J_{\text{top}}}{ne} = \frac{3.18\times10^6\ \text{A/m}^2}{(6.0\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})} = 3.31\times10^{-4}\ \text{m/s}$$

$$(v_{\text{d}})_{\text{mid}} = \frac{J_{\text{mid}}}{ne} = \frac{1.27\times10^7\ \text{A/m}^2}{(6.0\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})} = 1.33\times10^{-3}\ \text{m/s}$$

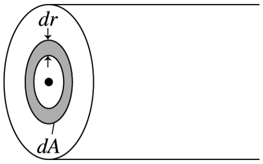
(e) The electron current is $i = N_{\text{e}}/\Delta t = I/e$. Because $I_1 = I_2 = I_3 = 10\ \text{A}$,

$$\left(\frac{N_{\text{e}}}{\Delta t}\right)_{\text{top}} = \left(\frac{N_{\text{e}}}{\Delta t}\right)_{\text{mid}} = \left(\frac{N_{\text{e}}}{\Delta t}\right)_{\text{bot}} = \frac{10\ \text{A}}{1.60\times10^{-19}\ \text{C}} = 6.25\times10^{19}\ \text{s}^{-1}$$

Quantity	Top	Middle	Bottom
I	10 A	10 A	10 A
J	3.2×10^6 A/m^2	1.27×10^7 A/m^2	3.27×10^6 A/m^2
E	0.091 V/m	0.36 V/m	0.091 V/m
v_d	3.3×10^{-4} m/s	1.33×10^{-3} m/s	3.3×10^{-4} m/s
τ	2.1×10^{-14} s	2.1×10^{-14} s	2.1×10^{-14} s
i	6.3×10^{19} s^{-1}	6.3×10^{19} s^{-1}	6.3×10^{19} s^{-1}

27.65. Model: The current density is modeled as $J(r) = kr$.

Visualize:



Solve:

$$I = \int dI = \int J\,dA = \int_0^R kr(2\pi r)dr = 2\pi k\int_0^R r^2 dr = 2\pi k[\tfrac{1}{3}kr^3]_0^R = \tfrac{2}{3}\pi kR^3 \Rightarrow k = \frac{3}{2}\frac{I}{\pi R^3}$$

Assess: The dimensions of the answer are right.

27.66. Model: The conductivity is modeled as $\sigma(r) = \sigma_0 - cr$.

Visualize: We are given $r = 0.30$ mm, $\sigma_0 = 5.0\times10^7\ \Omega^{-1}\ \text{m}^{-1}$ and $c = 1.2\times10^{11}\ \Omega^{-1}\ \text{m}^{-2}$. Consider the wire as nested cylindrical shells, each 4.0 m long.

Solve: The easiest approach is to find the reciprocal of the resistance: $G = 1/R$ where G is called the conductance and $G = \sigma A/L$ and dG is the conductance of one shell.

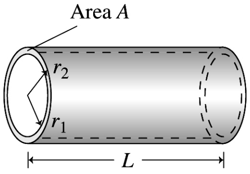
$$\frac{1}{R} = G = \int dG = \int_0^{r_0}(\sigma_0 - cr)\left(\frac{dA}{L}\right) = \frac{1}{L}\int_0^{r_0}(\sigma_0 - cr)(2\pi r)dr = \frac{2\pi}{L}\int_0^{r_0}(\sigma_0 r - cr^2)dr$$
$$= \frac{2\pi}{L}\left[\tfrac{1}{2}\sigma_0 r_0^2 - \tfrac{1}{3}cr_0^3\right] = \frac{2\pi}{L}r_0^2\left[\tfrac{1}{2}\sigma_0 - \tfrac{1}{3}cr_0\right]$$
$$= \frac{2\pi}{(4.0\text{ m})}(0.30\text{ mm})^2\left[\tfrac{1}{2}(5.0\times10^7\ \Omega^{-1}\text{m}^{-1}) - \tfrac{1}{3}(1.2\times10^{11}\ \Omega^{-1}\ \text{m}^{-2})(0.30\text{ mm})\right] = 1.84\ \Omega^{-1}$$

The resistance is the inverse of the conductance: $R = 1/1.84\ \Omega^{-1} = 0.54\ \Omega$.

Assess: A confirming way to think about it is that each of the cylindrical shells has a resistance dR, but the shells are in *parallel* with each other, so $\dfrac{1}{R} = \int \dfrac{1}{dR}$ which leads to the same result above.

27.67. Model: Assume the battery is ideal.

Visualize:



Solve: The tube's resistance is

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(r_2^2 - r_1^2)} = \frac{(1.5\times10^{-6}\ \Omega\text{m})(0.20\text{ m})}{\pi\left[(0.0015\text{ m})^2 - (0.0014\text{ m})^2\right]} = 0.329\ \Omega$$

When connected to the battery, $\Delta V_{\text{tube}} = \Delta V_{\text{bat}} = 3.0$ V. Thus, the current is

$$I = \frac{\Delta V_{\text{tube}}}{R} = \frac{3.0\text{ V}}{0.329\ \Omega} = 9.1\text{ A}$$

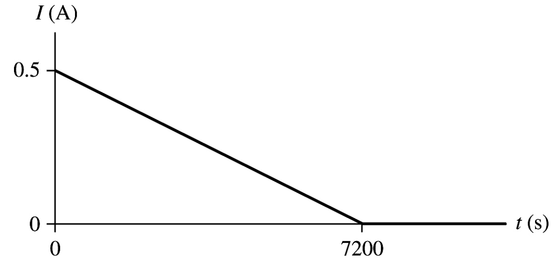
27.68. Model: Assume the resistors are ohmic.

Solve: The potential difference across the resistor on the left is ε, so the current in it is $I = \varepsilon/R$. The potential difference across the resistor on the right is 2ε. For the current to be the same in that resistor, the resistance must be twice as much, so that $I = 2\varepsilon/(2R)$. Therefore, the resistance of the resistor on the right is $2R$.

Assess: The two resistors are neither in parallel nor in series.

27.69. Model: Assume the battery is ideal.

Visualize: The current supplied by the battery and passing through the wire is $I = \Delta V_{\text{bat}}/R$. A graph of current versus time has exactly the same shape as the graph of ΔV_{bat} with an initial value of $I_0 = (\Delta V_{\text{bat}})_0/R = (1.5\text{ V})/(3.0\ \Omega) = 0.50\text{ A}$. The horizontal axis has been changed to seconds.



Solve: Current is $I = dQ/dt$. Thus the total charge supplied by the battery is

$$Q = \int_0^\infty I\,dt = \text{area under the current-versus-time graph}$$
$$= \tfrac{1}{2}(7200\text{ s})(0.50\text{ A}) = 1.80\times10^3\text{ C}$$

27.70. Model: The charged metal plates form a parallel-plate capacitor.

Solve: (a) When the charged plates are connected, the maximum current in the wire can be obtained from $I_{\text{max}} = \Delta V_{\text{max}}/R$ where ΔV_{max} is the potential difference across the capacitor just before the wire is connected. ΔV_{max} for the capacitor is $\Delta V_{\text{max}} = Q/C$, where C is the capacitance for a parallel-plate capacitor. Noting that $C = \varepsilon_0 A/d$ and $R = \rho L/\pi r^2$, the maximum current is

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{R} = \frac{Q}{CR} = \frac{Q}{(\varepsilon_0 A/d)(\rho L/\pi r^2)} = \frac{Q}{\varepsilon_0 \rho}\frac{\pi r^2}{A}$$
$$= \frac{(12.5\times10^{-9}\text{ C})\pi(0.112\times10^{-3}\text{ m})^2}{(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(1.7\times10^{-8}\ \Omega\text{ m})\pi(5.0\times10^{-2}\text{ m})^2} = 4.2\times10^5\text{ A}$$

In the above calculations, note that the length of the wire L is the same as the separation of the two plates d. Also, the resistivity of copper is taken from Table 27.2.

(b) The current I will decrease in time. As charge leaves one plate and moves to the other plate, the voltage across the capacitor and hence the current goes down.

(c) The energy dissipated in the wire is the energy present in the capacitor before it was connected with the wire. This energy is

$$U_{\text{dissipated}} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\frac{Q}{C} = \frac{1}{2}Q\Delta V_{\text{max}} = \frac{1}{2}(12.5\times10^{-9}\text{ C})(1800\text{ V}) = 1.1\times10^{-5}\text{ J}$$

27.71. Model: The volume of the wire remains constant as it is stretched. The cross-sectional area of the wire changes uniformly as it stretches.

Solve: The resistance of the wire before it is stretched is

$$R = \frac{\rho L}{A} = \frac{\rho L^2}{AL} = \frac{\rho L^2}{V}$$

The volume V remains constant as the wire is stretched. After stretching, the resistance is

$$R' = \frac{\rho L'^2}{V}$$

Taking the ratio of these two equations and using the fact that ρ is a property of the material and therefore does not change,

$$\frac{R}{R'} = \frac{L^2}{L'^2} = \frac{L^2}{(2L)^2} = \frac{1}{4} \Rightarrow R' = 4R$$

The wire's resistance is $4R$.

Assess: Stretching a wire increases the one dimension of length but decreases the two dimensions of cross-sectional area, so the resistance increases.

27.72. Solve: (a) The charge delivered is

$$(50\times10^3\ \text{A})(50\times10^{-6}\ \text{s}) = 2.5\ \text{C}.$$

(b) The current in the lightning rod and the potential drop across it are related by Equation 27.21. Using ρ for iron from Table 27.2,

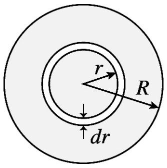
$$I = \frac{A}{\rho L}\Delta V \Rightarrow A = \frac{\rho LI}{\Delta V} = \frac{(9.7\times10^{-8}\ \Omega\text{m})(5.0\ \text{m})(50\times10^3\ \text{A})}{100\ \text{V}} = 2.43\times10^{-4}\ \text{m}^2$$

This is the area required for a maximum voltage drop of 100 V. The corresponding diameter of the lightning rod is

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{2.43\times10^{-4}\ \text{m}^2}{\pi}} = 1.8\times10^{-2}\ \text{m} = 1.8\ \text{cm}$$

Challenge Problems

27.73. Visualize:



The current density in the beam increases with distance from the center. We consider a thin circular shell of width dr at a distance r from the center to calculate the current density at the edge.

Solve: (a) The beam current is 1.5 mA. This means the beam transports a charge of $1.5\times10^{-3}\ \text{C}$ in 1 s. The number of protons delivered in 1 second is

$$\frac{1.5\times10^{-3}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 9.375\times10^{15} = 9.4\times10^{15}$$

(b) From $J = I/A$, the current in the ring of width dr at a distance r from the center is

$$dI = JdA = J_{\text{edge}}\left(\frac{r}{R}\right)(2\pi r dr) = J_{\text{edge}}\frac{2\pi r^2 dr}{R}$$

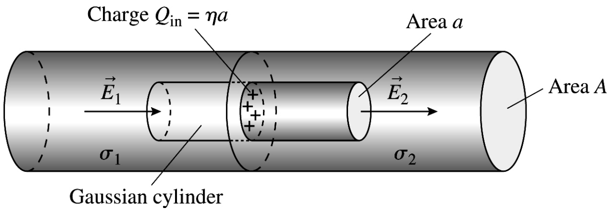
The total current $I = 1.5$ mA is found by integrating this expression:

$$I_{\text{total}} = \int dI = 1.5\times10^{-3}\ \text{A} = \frac{J_{\text{edge}}}{R}\int_0^R 2\pi r^2 dr = \frac{2\pi J_{\text{edge}} R^2}{3}$$

$$\Rightarrow J_{\text{edge}} = (1.5\times10^{-3}\ \text{A})\frac{3}{2\pi(2.5\times10^{-3}\ \text{m})^2} = 115\ \text{A/m}^2$$

27.74. Model: The currents in the two segments of the wire are the same.

Visualize: The electric fields $\vec{E}_1$ and $\vec{E}_2$ point in the direction of the current. Establish a cylindrical Gaussian surface with end area *a* that extends into both segments of the wire.



Solve: (a) Because current is conserved, $I_1 = I_2 = I.$ The cross-section areas of the two wires are the same, so the current densities are the same: $J_1 = J_2 = I/A.$ Thus the electric fields in the two segments have strengths

$$E_1 = \frac{J_1}{\sigma_1} = \frac{I}{A\sigma_1} \qquad E_2 = \frac{J_2}{\sigma_2} = \frac{I}{A\sigma_2}$$

The electric field enters the Gaussian surface on the left (negative flux) and exits on the right. No flux passes through the wall of cylinder, so the net flux is $\Phi_e = E_2 a - E_1 a.$ The Gaussian cylinder encloses charge $Q_{\text{in}} = \eta a$ on the boundary between the segments. Gauss's law is

$$\Phi_e = \frac{Q_{\text{in}}}{\varepsilon_0} \Rightarrow E_2 a - E_1 a = \frac{Ia}{A}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right) = \frac{\eta a}{\varepsilon_0}$$

Thus the surface charge density on the boundary is

$$\eta = \frac{\varepsilon_0 I}{A}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

(b) From the expression obtained in part (a)

$$\eta = \frac{Q}{\pi R^2} = \frac{I\varepsilon_0}{(\pi R^2)}\left(\frac{1}{\sigma_{\text{iron}}} - \frac{1}{\sigma_{\text{copper}}}\right)$$

$$\Rightarrow Q = (5\ \text{A})(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)\left(\frac{1}{1.0\times10^7\ \Omega^{-1}\text{m}^{-1}} - \frac{1}{6.0\times10^7\ \Omega^{-1}\text{m}^{-1}}\right) = 3.7\times10^{-18}\ \text{C}$$

Assess: This charge corresponds to a deficit of a mere 23 electrons on the boundary between the metals.

27.75. Model: The conductivity is modeled as $\sigma(r) = \sigma_0 - cr.$

Visualize: We are given $R = 5000\,\Omega$ and $C = 300\ \mu F.$

Solve: The current through the resistor is the negative of the rate at which charge is leaving the capacitor: $I_R = -dQ_C/dt,$ but the charge on the capacitor is $Q_C = C(\Delta V_C).$ From Ohm's law we also have $I_R = -V_C/R.$

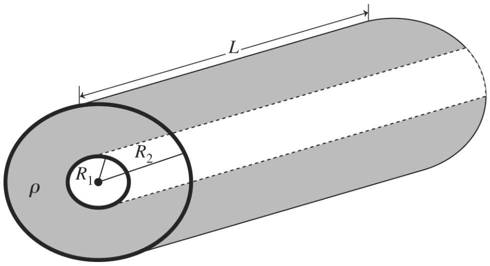
$$\frac{\Delta V_C}{R} = I_R = -\frac{dQ_C}{dt} = -C\frac{d(\Delta V_C)}{dt} \Rightarrow \frac{d(\Delta V_C)}{\Delta V_C} = -\frac{dt}{RC}$$

Integrate both sides with appropriate limits.

$$\int_{(\Delta V_C)_0}^{\frac{1}{2}(\Delta V_C)_0}\frac{d(\Delta V_C)}{\Delta V_C}=-\int_0^t\frac{dt}{RC}\Rightarrow \ln\frac{\frac{1}{2}(\Delta V_C)_0}{(\Delta V_C)_0}=-\frac{t}{RC}\Rightarrow t=-RC\ln\frac{1}{2}=-(5000\ \Omega)(300\ \mu\text{F})\ln\left(\frac{1}{2}\right)=1.0\text{ s}$$

Assess: Note that the result does not depend on the original ΔV of the battery; as long as we seek the time for the capacitor voltage to decrease by half, the answer is the same.

27.76. Model: Model the device as a series of nested cylindrical shells, each with resistance *dR*.
Visualize: Because the current is flowing radially we think of each shell as having a "length" (thickness) *dr* and an area $A = 2\pi rL$.



Solve: Because the shells are in series the total resistance is simply the sum of the resistances.

$$R=\int dR=\int\rho\frac{dr}{A}=\int\rho\frac{dr}{2\pi rL}=\frac{\rho}{2\pi L}\int_{R_1}^{R_2}\frac{dr}{r}=\frac{\rho}{2\pi L}\left[\ln r\right]_{R_1}^{R_2}=\frac{\rho}{2\pi L}\ln\left(\frac{R_2}{R_1}\right)$$

Assess: We expected the resistance to depend directly on the resistivity and inversely on L.